

Classicalization of Quantum States Induced by Amplification Process

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It has been shown that a macroscopic quantum system being in a high-temperature thermal coherent state can, in principle, be driven into non-classical states by coupling it to a microscopic quantum system. Therefore, thermal coherent states do not truly represent a purely classical system such as a measurement apparatus. Here, we investigate the classical limit of the quantum description of a more relevant macroscopic quantum system, namely a phase-preserving linear amplifier. In particular, we examine to what extent it is possible to find an amplified coherent state, supposedly representing the pointer state of a detector, in a superposition state, by coupling it to a qubit system. We demonstrate quantitatively that the classicalization induced by a realistic amplifier might not be isomorphic to that of a high-temperature thermal coherent state, offering a route to a classical state in a sense of not being able to be projected into a macroscopically distinct superposed state.

I. INTRODUCTION

Decoherence theory, by invoking the notion of environment, attempts to give an account of how the classical behavior emerges from the quantum-mechanical description [1, 2]. But, prior to that, the question is how can we characterize the classical limit of a quantum state in the first place? As Bell said: “What exactly qualifies some physical systems to play the role of measurer?” [3]. More relevant to the present work, we refine this question: what exactly qualifies a *quantum* state to represent the physical state of an individual macroscopic object, e.g. a measurement apparatus or a cat for that matter after the interaction with the environment?

Since the advent of quantum mechanics, there have been several proposals as to how the classical behavior is emerged from the underlying quantum dynamics (for a rather complete review see [4]). The first one was proposed by Bohr, via his *correspondence principle*, which states that classical behavior emerges from quantum description in the limit of large quantum numbers [5]. But, it turned out that this is not the case in general. As a counterexample, an oscillator in a large number state doesn’t faithfully represent the classical behavior. This motivated Schrödinger to propose an alternative quantum description of a “classical-like” state of an oscillator, the so-called “coherent state”, whose dynamics closely resembles that of a classical one [6].

Despite the promising classical features of coherent states, because of the linearity of the Schrödinger equation, a system which is in a coherent state can be driven into a superposition state by being coupled to a microscopic quantum system [7–15]. In fact, the non-classical features of coherent states are even recognized to be useful in the applications of quantum information science [16, 17]. The problem remains even if we consider a more noisy state, such as a high-temperature thermal coherent state [18–22]. Furthermore, it has been demonstrated that the violation of Leggett-Garg inequality can be achieved even for high-temperature thermal coherent

states [23]. Therefore, the thermalization process by itself is not sufficient to make a quantum state classical.

In the early 1980s, it was suggested that the *amplification process* in detectors might have a crucial role in the quantum-to-classical transition [24–27]. The development of MASERS as possible amplifiers triggered a flurry of interest in the quantum description of amplification process in 1960s [28–32], which led to the realization that a linear amplifier unavoidably adds noise to the input signal. This fundamental limit is expressed formally as a bound on the second moment of the added noise [33, 34]. In this context, quantum non-demolition measurements are designed to circumvent the limitations imposed by such limit when performing repeated measurements of quantum states [35–37]. The quantum limit on the entire distribution of the added noise was provided just recently [38]. Notably, a realistic amplifier transforms the quantum state of a detector into a form which is not equivalent to the Gaussian distribution of thermal coherent states. An amplified state is supposed to represent the pointer state from which the measurement result is read-out, and which we expect to have a definite value at the macroscopic scale. Therefore, we believe that it is timely to revisit this problem in the light of the recent advances in the quantum amplifiers and see if the quantum description of a realistic amplifier yields the most “classical-like” quantum state.

In this work, we analyze the classicality of the coherent states which have undergone the amplification process, by looking at the possibility of projecting them into superposition states (see FIG. 3). In particular, we focus on a mathematical model put forward by Caves and co-workers to analyze phase-preserving linear amplifiers [38]. In essence, the amplification of the input mode requires it to be coupled to an external mode, called ancillary mode, which adds noise to the output mode. The state of the ancillary mode determines the effect of the added noise on the input state. In an ideal linear amplifier, the ancillary mode is in the vacuum state. Our results show that while an ideal linear amplifier cannot reproduce a clas-

sical state, a realistic (non-ideal) one is able to produce states having vanishing non-classical features.

II. BASIS AMBIGUITY: STATING THE PROBLEM

A macro-realistic state accounts for a single macroscopic definite value at a given time, regardless of which basis or representation we choose for its description (see [39] for detailed discussion). Is it then accurate to say that an object in a coherent state $|\alpha\rangle$ has macro-realistic character and therefore is a valid representation of, e.g. the pointer's state of a measurement apparatus? From quantum-mechanical point of view, it is equally valid to think of it as

$$|\alpha\rangle \equiv \frac{1}{\sqrt{2}}(|\psi^+\rangle + |\psi^-\rangle) \quad (1)$$

by changing the basis in which $|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle \pm e^{i\varphi} |-\alpha\rangle)$, for $\langle\alpha|-\alpha\rangle \approx 0$ are Schrödinger's cat states, and therefore the system being in a coherent state can be found, in principle, in a cat state regardless of the feasibility issue. Along this argument, a classical mixture

$$\frac{1}{2}|\alpha\rangle\langle\alpha| + \frac{1}{2}|-\alpha\rangle\langle-\alpha|, \quad (2)$$

which is considered as the superposition-free final state *after the interaction with the environment*, is also incompatible with the macroscopic definiteness. Because, it can be equally well represented by

$$\frac{1}{2}|\psi^+\rangle\langle\psi^+| + \frac{1}{2}|\psi^-\rangle\langle\psi^-|. \quad (3)$$

which is a statistically inequivalent mixture. A priori quantum mechanics is unbiased toward any of these two inequivalent representations; as there is no unique ensemble decomposition of the mixed states. The same objection applies even to a thermal coherent state (e.g. see [19]). However, the classical limit of a quantum state should have a unique representation, independent of the basis of the quantum state. Therefore, considering coherent states, or even thermal coherent states, as classical limit of quantum states are untenable. This problem is termed as the *basis ambiguity problem* [2, 40], and in our opinion it is not satisfactorily resolved yet.

A compelling resolution to this problem, from our point of view, is that the classical limit of a quantum state is a state which contains a sufficient amount of noise, hindering a generation of entanglement with a genuine microscopic quantum state, and yet it has a well-defined pointer position in the coarse-grained macroscopic scale, which is capable of being as a meter.

Here, we look at this scenario and pay a particular attention to the quantum state of the macroscopic output signal, namely, the position of a pointer revealing

the value of a measurement outcome. This output signal is generated by the amplification process of an initial microscopic signal. An important fact to bear in mind is that the macroscopic (or classical) signal does not reveal the exact microscopic details of the state. This means that there might be many microscopic states, in the fine-grained scale, basically producing the same classical signal of the pointer state. Therefore, the quantum state of the output signal is represented by a probabilistic mixture of all these consistent microscopic states (see FIG. 1). Here, since the classical signal does not distinguish between the micro-states, we assign equal probability to them. This produces a top-flatted probability distribution. This consideration is in accordance with the idea that coarse-grained or unsharp measurements give rise to classical behavior [41]. The associate effect (or Kraus) operators of an unsharp observation determine the post-measurement state given by the Lüder rule [42]. The post-measurement state is accordingly a noisy state.

The recent advances on quantum description of real-

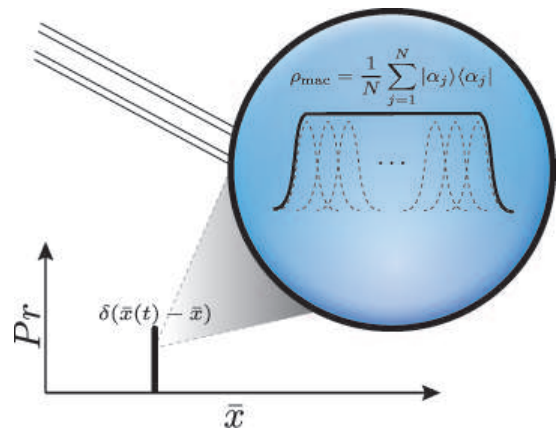


FIG. 1. The pointer's state, denoted by ρ_{mac} , has a well-defined position \bar{x} at the coarse-grained scale. However, a sharp measurement resolves different microscopic states which are consistent with the same pointer's macroscopic state.

istic amplifiers, motivated us to look at the classicality of amplified states. Amplification process not only amplifies the input signal but also adds noise to the input state, producing a macroscopic signal which is noisy at the microscopic scale, and yet having well-defined pointer position state at the coarse-grained macroscopic scale. It is worth mentioning that the resulting amplified state is not in an equivalent form to a thermal coherent state. Therefore, it is intriguing to see if, unlike a thermal coherent state, its probability of being found in a cat state, vanishes. Before investigating this problem let us briefly review the amplification process.

III. AMPLIFICATION PROCESS

The setting for our analysis is a bosonic mode \hat{a} , called *primary* mode, which is to undergo amplification pro-

cess. The type of amplification one typically thinks of in physics is *linear* amplification, which means that the output mode is linearly related to the input mode (e.g. being multiplied by some fixed amplitude gain g). Here, we shall deal with linear amplification, since it is straightforward to treat mathematically. Also we wish to amplify both quadratures of the input mode with the same gain. This type of amplification is often referred to as *phase-preserving* amplification (FIG. 2). A *perfect* phase-preserving linear amplifier transforms the input mode directly to the output one: $\hat{a}_{out} = g\hat{a}_{in}$. However, this transformation violates unitarity. Physically, this means that amplification of the primary mode requires it to be coupled to an external mode \hat{b} , called ancillary mode, which adds noise to the output mode. When referred to the input, the output noise is constrained as $\langle |\Delta a_{out}|^2 \rangle / g^2 \geq \langle |\Delta a_{in}|^2 \rangle + 1/2$ [33]. The minimum added noise, corresponding to the lower bound, is the half-quantum of vacuum noise. The amplifier working with the minimum added noise is called an *ideal* amplifier. The simplest model of such an amplifier is provided by a parametric amplifier [43–45]. The ideal amplified state of the input state ρ is given by [38]

$$\varepsilon(\rho; g) = \text{tr}_b(\hat{S}\rho \otimes \sigma \hat{S}^\dagger), \quad (4)$$

where $\hat{S} = e^{r(\hat{a}\hat{b} - \hat{a}^\dagger\hat{b}^\dagger)}$ is the two-mode squeezing operator, with the amplitude gain being $g = \cosh r$, and σ is the positive density operator of the ancillary mode. The density operator σ is diagonal in the number basis

$$\sigma = \sum_{n=0}^{\infty} \lambda_n |n\rangle\langle n|, \quad (5)$$

where ‘ λ_n ’s are the corresponding eigenvalues. Note that for an ideal amplifier, we have $\sigma = |0\rangle\langle 0|$.

The amplified state for the complete distribution of the added noise is given by the amplifier map [38]

$$\varepsilon(\rho; g) = \hat{B}(\hat{A}(g)\rho). \quad (6)$$

The superoperator \hat{A} amplifies the input state ρ with the amplitude gain g . For a coherent input state, the output of $\hat{A}(g)$ is just a displaced coherent state: $\hat{A}(g)(|\alpha\rangle\langle\alpha|) = |g\alpha\rangle\langle g\alpha|$. The superoperator \hat{B} adds a noise to the output state by smearing out a phase-space distribution into a broader distribution

$$\hat{B} = \int d^2\beta \Pi^{-1}(\beta) \hat{D}(\hat{a}, \beta) \odot \hat{D}^\dagger(\hat{a}, \beta), \quad (7)$$

where \odot marks the slot where the input to the superoperator goes and $\hat{D}(\hat{a}, \beta)$ is the displacement operator for the mode \hat{a} ; $\hat{D}(\hat{a}, \beta) = e^{\beta\hat{a}^\dagger - \beta^*\hat{a}}$. The real-valued function $\Pi^{-1}(\beta)$ is called the *smearing function*. This function is independent of the input state, but it depends on the gain g as

$$\Pi^{-1}(\alpha) = \frac{e^{-|\alpha|^2/g^2-1}}{\pi(g^2-1)} \sum_{n=0}^{\infty} \frac{\lambda_n |\alpha|^{2n}}{n!} (g^2-1)^n. \quad (8)$$

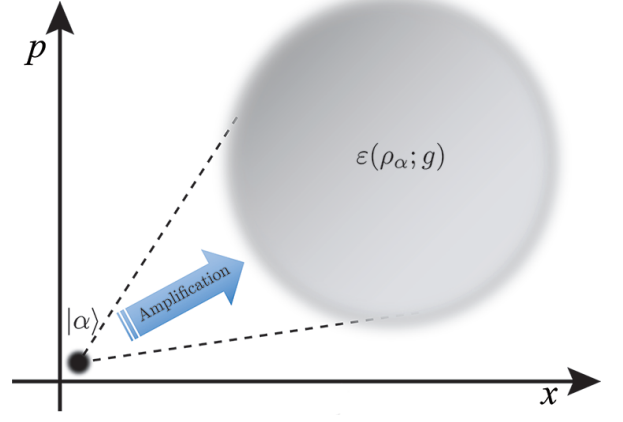


FIG. 2. The initial coherent state undergoes linear phase-preserving amplification, resulting the smearing of the probability distribution in phase space.

Note that the smearing function of an ideal linear amplifier is isomorphic to the Glauber-Sudarshan P function of the thermal coherent state $P_{th}(v, d) = \frac{2}{\pi(v-1)} \exp[-\frac{2|\alpha-d|^2}{v-1}]$, where v is the variance and d is the displacement in the phase-space. We safely assume that before amplification due to the internal dynamics of the macroscopic system, the system is evolved to the most classical pure state, i.e. a coherent state $\rho_\alpha = |\alpha\rangle\langle\alpha|$.

Our analysis here is mathematically based on the optical states. Nonetheless, the generalization of our approach to the corresponding mechanical states is straightforward. For example, a superconducting qubit can manifest macroscopic distinguishable states by injecting currents of opposite verses. The corresponding wave functions are effectively Gaussian states. They are isomorphic to the coherent states in our analysis.

IV. A CASE STUDY: AN AMPLIFIER INTERACTING WITH A TWO-LEVEL SYSTEM

The amplified coherent state is obtained as

$$\varepsilon(\rho_\alpha; g) = \int d^2\beta \Pi^{-1}(\beta - g\alpha) |\beta\rangle\langle\beta|. \quad (9)$$

We consider $\varepsilon(\rho_\alpha; g)$ as the initial state of the detector, interacting with a qubit system, as illustrated in Fig. 3. The qubit is prepared in state $|+\rangle = (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$, which is the eigensate of the x component of Pauli operator, and $|\uparrow\rangle$ and $|\downarrow\rangle$ are the spin-up and spin-down states of the qubit in the z direction. Qubit being in $|+\rangle$ interacts with an amplified coherent state $\varepsilon(\rho_\alpha; g)$, with the interaction Hamiltonian being $c\hat{a}^\dagger\hat{a}|\uparrow\rangle\langle\uparrow|$, where c is the coupling strength. After $t = \pi/c$, the resulting state

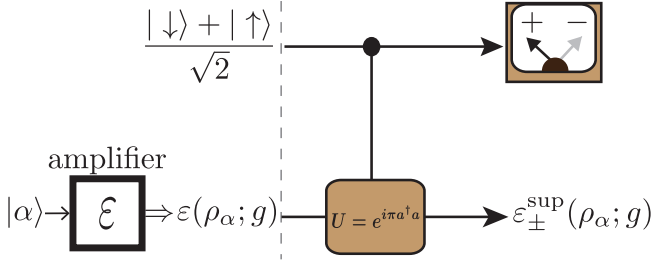


FIG. 3. Circuit for projecting the amplified state to a non-classical state, by coupling it to a two-level system.

is

$$\begin{aligned}
 & U_\pi (|+\rangle\langle+| \otimes \varepsilon(\rho_\alpha; g)) U_\pi^\dagger \\
 &= |+\rangle\langle+| \otimes E_+ \varepsilon(\rho_\alpha; g) E_+^\dagger + |-\rangle\langle-| \otimes E_- \varepsilon(\rho_\alpha; g) E_-^\dagger \\
 &+ |+\rangle\langle-| \otimes E_+ \varepsilon(\rho_\alpha; g) E_-^\dagger + |-\rangle\langle+| \otimes E_- \varepsilon(\rho_\alpha; g) E_+^\dagger.
 \end{aligned} \quad (10)$$

where $E_\pm = (\mathbb{1} \pm U_\pi)/2$ and $U_\pi = \exp(i\pi a^\dagger a)$. Upon the qubit measurement on the basis $|\pm\rangle$, the detector's state is projected into a superposition state

$$\varepsilon_\pm^{\text{sup}}(\rho_\alpha; g) = \frac{E_\pm \varepsilon(\rho_\alpha; g) E_\pm^\dagger}{p_\pm} \quad (11)$$

conditioned on the result of the qubit measurement. Here, we have defined

$$\begin{aligned}
 E_\pm \varepsilon(\rho_\alpha; g) E_\pm^\dagger &= \int d^2\beta \Pi^{-1}(\beta - g\alpha) \{ |\beta\rangle\langle\beta| + |-\beta\rangle\langle-\beta| \\
 &\pm |\beta\rangle\langle-\beta| \pm |-\beta\rangle\langle\beta| \}, \quad (12)
 \end{aligned}$$

and $p_\pm = \text{Tr}\{E_\pm \varepsilon(\rho_\alpha; g) E_\pm^\dagger\}$ is the probability of finding the amplifier in the corresponding superposition state, $\varepsilon_\pm^{\text{sup}}(\rho_\alpha; g)$. Note that this setting is basically equivalent to the case mentioned in the preceding section for examining the basis ambiguity problem.

The probability distribution of diagonal and off-diagonal elements of $\varepsilon^{\text{sup}}(\rho_\alpha; g)$ can be obtained as $Pr(x) = \langle x | \varepsilon^{\text{sup}}(\rho_\alpha; g) | x \rangle$ and $Pr(p) = \langle p | \varepsilon^{\text{sup}}(\rho_\alpha; g) | p \rangle$, respectively. The two peaks along $x (\equiv \text{Re } \alpha)$ axis are well-separated and represent the pointer positions, if the measurement has been performed in $\{|\uparrow\rangle, |\downarrow\rangle\}$ basis. Interference fringes along $p (\equiv \text{Im } \alpha)$ axis are a typical signature of quantum superposition between macroscopically distinct states. It is worth mentioning that the interference pattern indicates the generated quantum entanglement between the qubit and the amplified mode proceeding the projective measurement on the qubit. Therefore, any amplified state truly representing the classical limit should suppress the generated entanglement with the qubit. The amplitude and the pattern of peaks and interference fringes depend on the choice of ancillary eigenvalues λ_n . The only constraint imposed by quantum mechanics is to guarantee that σ is a valid density operator, λ_n s should be non-negative [46]. It is

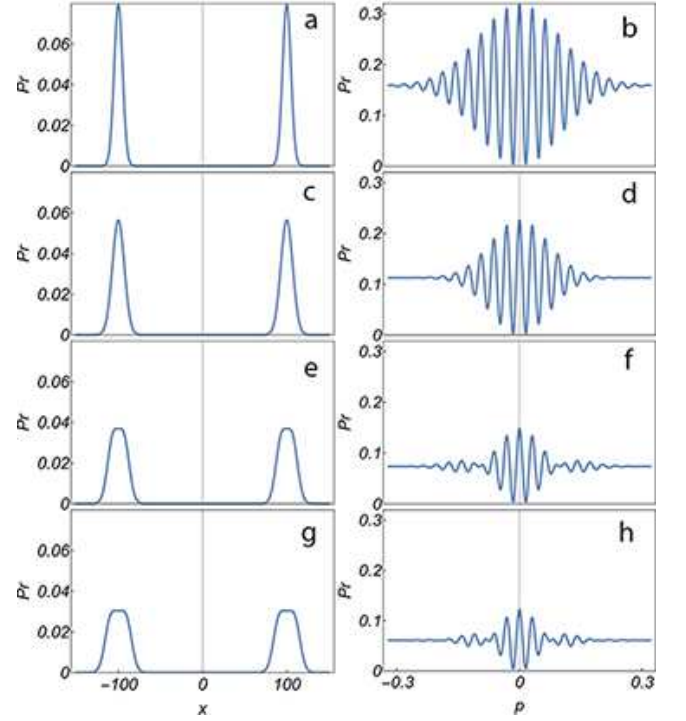


FIG. 4. The probability of x (left) and probability of p (right) for a high-temperature thermal coherent state with $d = v = 100$ (a,b), an amplified coherent state with $g = \alpha = 10$ for the first term (c,d), for the first two terms with $\lambda_1 = 0.3$, $\lambda_2 = 0.7$ (e,f), and for the first three terms with $\lambda_1 = 0.2$, $\lambda_2 = 0.3$ and $\lambda_3 = 0.5$ (g,h). It is obvious that as we add the non-ideal effects to the ideal linear amplifier, the interference fringes are weakened gradually.

not yet clear in detail how λ_n s are parameterized in actual “non-ideal” amplifiers. Nonetheless, we found that the most appropriate choice to ensure the emergence of classicality is $0 < \lambda_n < \lambda_{n+1}$. The peaks and interference fringes for a high-temperature thermal coherent state and the corresponding amplified coherent states for ancillary mode with one, two and three available states are plotted in FIG. 4, for an optimized choice of λ_n s. Notably, the non-ideal amplification process for certain range of parameters produces a probability distribution (see Fig. 4(e,g)) which has similar top-flattened shape with that of our heuristic model illustrated in FIG. 1.

According to FIG. 4, as we include the non-ideal terms in the ideal phase-preserving linear amplifier, the interference effects are weakened gradually. This also can be verified, using a quantitative measure of macroscopicity. Lee and Jeong introduced a general and inclusive measure of macroscopicity in the phase space as [47]

$$S(\rho) = \frac{\pi^M}{2} \int d^2\alpha W(\alpha) \sum_{m=1}^M \left[-\frac{\partial^2}{\partial \alpha_m \partial \alpha_m^*} - 1 \right] W(\alpha), \quad (13)$$

where $W(\alpha)$ is the Wigner function of state and M is the number of modes. The measure $S(\rho)$ is plotted for

amplified coherent states with the first term, with first two terms and with first three terms in FIG. 5. As we expected, this shows that with a large gain g , as we include the non-ideal terms in the ideal amplifier, the probability of macroscopic quantum superpositions decreases.

Let us come back to the problem of thermal coher-

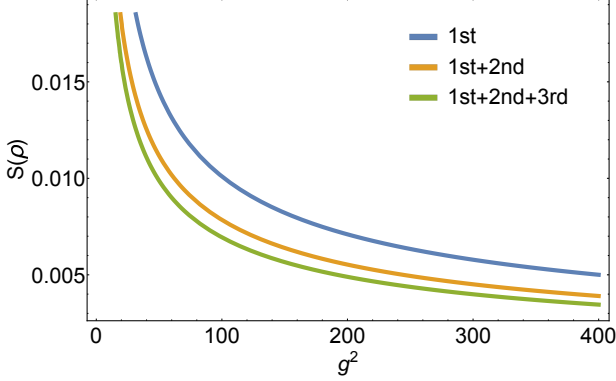


FIG. 5. Interference-base measure $S(\rho)$ for a linear amplifier with $\alpha = 10$ for the first term (blue), for two first terms with $\lambda_1 = 0.3$, $\lambda_2 = 0.7$ (orange), for three first terms with $\lambda_1 = 0.2$, $\lambda_2 = 0.3$, $\lambda_3 = 0.5$ (green). This shows that the non-ideal linear amplifier with large gain g is able to produce a non-classical state.

ent state versus amplified coherent state. We choose a temperature for the initial thermal coherent state, which gives rise to the same amount of the interference suppression, as the amplified coherent state with three terms in FIG. 4,h. We depicted the pointer positions and the corresponding interference fringes of the required thermal coherent state in FIG. 6. We observe that the Gaussian picks spread a bit more, compared with those of the amplified states, showing qualitatively different probability distributions.

To ensure that the non-ideal linear amplifier has a non-thermal effect, we compare the interference fringes appeared in the high-temperature thermal coherent state with those of a non-ideal amplified coherent state with equal purity, $\text{Tr}(\rho_{th}^2) = \text{Tr}(\{\varepsilon(\rho_\alpha; g)\}^2)$. For a thermal state with $v = d = 100$, the amplifier gain g of the corresponding amplified coherent state with the first term, the first two terms and the first three terms are 7.10, 5.28 and 4.56, respectively. For a fixed purity, the interference fringes for an amplifier with the corresponding terms are plotted in FIG. 7. The suppression of interference shows that including non-ideal terms to an ideal amplifier has a non-thermal effect. One can imagine that the inclusion of many of such terms in the actual detector completely vanishes interference.

In general, the ancillary mode can be found in a classical or non-classical state. Of course, as a part of the detector it is more likely to be found in a classical state. Therefore, we examine the case in which the ancillary mode is prepared in a thermal coherent state. According to FIG. 8, the pointer positions of the corresponding am-

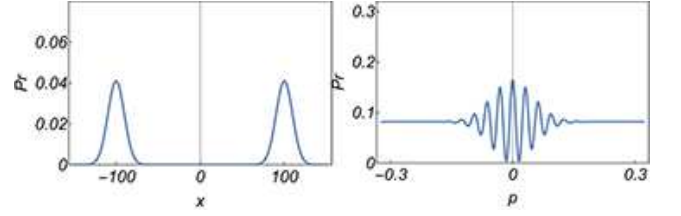


FIG. 6. The probability of x and p for a thermal coherent state with $d = 100$ and $v = 380$. The probability of p at $p = 0$ is equal to that of the amplified coherent state with three terms, with $g = \alpha = 10$, plotted in FIG.2,h

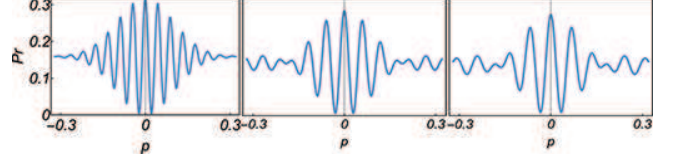


FIG. 7. The probability of p for an amplified coherent state including the first term with $g = 7.10$ (left), the first two terms with $\lambda_1 = 0.3$, $\lambda_2 = 0.7$ and $g = 5.28$ (middle) and the first three terms with $\lambda_1 = 0.2$, $\lambda_2 = 0.3$, $\lambda_3 = 0.5$ and $g = 4.56$ (right). The purity of these amplified states is equal to the purity of a thermal coherent state, with $v = d = 100$, which is 0.01.

plified coherent state do not have a top-flat shape, and thus it is not appropriate to produce a classical state. This is because of the fact that in a thermal coherent state the order of eigenvalues λ_n are decreasing, while for the emergence of classicality the appropriate order is increasing (see FIG. 4).

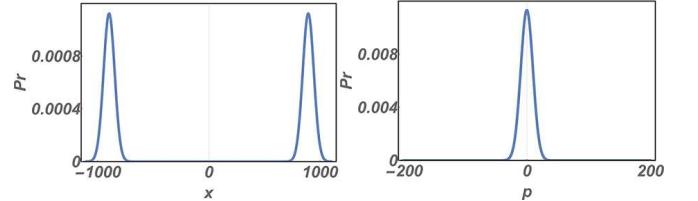


FIG. 8. The probability of x (left) and probability of p (right) of an amplified coherent state with $g = \alpha = 10$ for ancillary mode in a thermal coherent state with $d = v = 100$. We can clearly recognize that the pointer positions (left) do not have a top-flat shape.

V. DISCUSSION & CONCLUDING REMARKS

It is of significance to come up with a form of quantum state representing the state of a macroscopic system, and resolving the problem of basis ambiguity, without invoking the entanglement with the environment or modifying the laws of quantum mechanics. That is why, we look at the amplifier, as it is supposed to describe the pointer's state, and examine the basis ambiguity of the amplified

state via a non-classical evolution sketched in FIG. 1. Our results show that an amplifier state can demonstrate a behavior which is not similar to that of a thermal coherent state when it is coupled to a microscopic state, yet having well-defined pointer position state at the coarse-grained macroscopic scale. Nonetheless, we stress that to achieve a more conclusive result we need to examine a non-ideal amplifier state with realistic parameters. To

our best knowledge this is yet to be fully identified, and this issue requires further research.

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